Abstract—Salty sweat secretions in the epidermis change the skin's electrical activity resulting in the measured skin conductance signal. While the relatively fast variation of skin conductance (i.e., phasic component) reflects sympathetic nervous system activity, the slow variation (i.e., tonic component) is related to thermoregulation and general arousal. To better understand the neural information encoded in a skin conductance signal, it is necessary to decompose it into its constituent components. We model the fast variations using a second order differential equation incorporating a sparse impulsive input to the model. Furthermore, we model the tonic component with several cubic basis spline functions. Finally, we develop a block coordinate descent approach for skin conductance signal decomposition by employing generalized-cross-validation for balancing between smoothness of the tonic component, the sparsity of the neural stimuli, and residual error. We analyze experimental and simulated data to validate the performance of the proposed approach. We successfully illustrate its ability to recover the neural stimuli, the underlying physiological system parameters, and both tonic and phasic components. In summary, we develop a novel approach for decomposition of phasic and tonic components of skin conductance signal using a generalized-cross-validation-based block coordinate descent approach. Recovering the underlying neural stimuli and the tonic component accurately could potentially improve cognitive-stress-related arousal states estimation for better stress regulation in mental health disorders.

I. INTRODUCTION

According to the study by Walker et al. [1], 14.3% of deaths worldwide are attributable to mental health-related disorders. Identification of problematic patterns of emotional regulation could potentially help characterize psychiatric disorders [2]. Different physiological signals such as electroencephalogram, heart rate, respiration, and electrodermal activity (EDA) could be utilized to identify problematic patterns of emotional regulation [3]. Hence, a personalized mental health monitoring wearable system based on various physiological signals could eventually lead to effective regulation of mental health-related problems [4].

Any activity related to changes in the electrical characteristics of the epidermis is known as the EDA. Skin conductance (SC), a measure of EDA, is defined as the conductance measured across two distant points in the epidermis in the presence of salty sweat secretions produced by different eccrine sweat glands [5]. Depending on the physiological need, the autonomic nervous system (ANS) stimulates sweat glands to produce sweat which changes the conductivity of the epidermis [5], [6], [7]. These changes in conductivity can be analyzed to understand the underlying ANS activity, especially the sympathetic nervous activity, which contains a great deal of information about human arousal [8].

SC can be modeled as a summation of two components [5], [6], [7]. The relatively slow varying component, which is called the tonic component, is generally dependent on the thermoregulation of the body, ambient temperature, and humidity. This tonic component also contains information about the general arousal of a person [5]. On the other hand, the comparatively fast varying component, which is called the phasic component, is a reflection of neural stimulation from the sympathetic nervous system, a part of the autonomic nervous system (ANS) [5]. Hence, EDA can be represented as the sum of two convolution operations: (1) a sparse sympathetic nervous system brain activity and a fast physiological smoothing system, (2) a periodic activation and a slow physiological smoothing system.

Many SC decomposition strategies have been proposed by different researchers to recover the timings and amplitudes of neural stimulation as well as to estimate the underlying physiological parameters. Benedek et al. [7] proposed a non-negative deconvolution scheme called LedaLab for SC time series to separate them into discrete SC responses. However, this method leads to a non-sparse solution for neural stimuli which may overfit the noise. Moreover, their decomposition scheme does not include individual differences in the modeling of the physiological system parameters. Greco et al. [9] proposed a quadratic programming based decomposition approach called cvxEDA for SC while considering the sparsity condition in neural stimuli. However, the manual selection of the regularization parameter for imposing the sparsity constraint makes it challenging to find an appropriate sparsity level for the neural stimuli. Studies in [10], [11], [12] proposed coordinate descent deconvolution approaches to account for the individual differences in the physiological system parameters, but do not solve for the tonic component.

In the present study, we propose an algorithm to recover the ANS neural stimuli, the underlying physiological system parameters, and the tonic component from observed SC sampled data. Inspired by the work carried out by Faghih et al. [10], [13], [14], [15], [16] and Greco et al. [9], we use a second order differential equation model to relate
SC to the internal unobserved neural stimuli and model the tonic component with a set of cubic basis-spline (B-spline) functions. We propose a block coordinate descent approach to recover the unknowns by incorporating sparse recovery for the neural stimuli and the interior-point method for the physiological system parameters and the tonic component estimation. Moreover, we implement generalized-cross-validation (GCV) to obtain regularization parameters for both the $l_1$-norm and $l_2$-norm penalization terms in each iteration of the block coordinate descent approach. Finally, we analyze both experimental and simulated skin datasets to show the performance of our proposed approach.

II. METHOD

A. Experiment

In this study, we use the SC responses to loud sounds, simultaneously recorded from palm, fingers and foot data [17]. The experiment was performed to study and model event-related SC responses. Details of the experiment are provided in [18]. We use the SC recording from the middle phalanx of the dominant hands of two female and two male subjects. The sampling frequency for this data set is 100 Hz. Usually, the sampling frequency is kept much higher than the SC signal bandwidth to avoid aliasing from high frequency noise sources.

B. Model Formulation

The SC signal can be represented combining the phasic and tonic components as follows:

$$y(t) = y_p(t) + y_s(t)$$

where $y(t)$, $y_p(t)$ and $y_s(t)$ represent the SC signal, and the phasic and tonic components, respectively.

1) Phasic Component:

The phasic component can be considered as a smoothed version of neural spiking activity from the brain. We model this smoothing filter using the first order diffusion kinetics of sweat from the sweat ducts to the strata cornea and subsequent evaporation from the strata cornea [6], [7], [5]. We can combine both kinetics and use the following second order differential equation including the stimulation from sudomotor $u(t)$ to describe the phasic component in the sweat glands:

$$\tau_r \frac{d^2 y_p(t)}{dt^2} + \tau_r \frac{dy_p(t)}{dt} + y_p(t) = u(t)$$

where $u(t)$, $\tau_r$ and $\tau_d$ represents the neural stimuli generated by the sympathetic nervous system, the rise and decay times for each SC response, respectively. As the number of impulses in the neural stimuli is very small compared to the number of samples in the discrete signal, we can consider the neural stimuli as a sparse signal in our analysis similar to [11]. By letting $T_u$ be the sampling frequency of the neural stimuli, similar to [10], [13], [14], [15], [11], [12], we define an abstract definition of $u(t)$ as the summation of weighted and shifted impulse functions, i.e., $u(t) = \sum_{i=0}^{N} u_i \delta(t - iT_u)$, where $u_i$ represents the amplitude of the neural stimulus from ANS at time $iT_u$. $N$ is the number of samples in the discrete form of $u(t)$ and can be written as a function of the duration of the recorded signal $T_d (N = \frac{T_d}{T_u})$; $u_i$ is zero if there is no neural impulse or any positive value if there exists an impulse at time instance $iT_u$.

In order to obtain a solution to the system equation, we assume that initially the sweat duct is empty similar to [5], [10], [11], [12]. Hence, the solution to the differential equation becomes,

$$y_p(t) = y_p(0)e^{-\frac{t}{\tau_r}} + h(t) * u(t)$$

where $h(t)$ refers to the system response and can be represented as a scaled version of the Bateman function. Here, the operator ‘$*$’ represents the convolution operation. $h(t)$ can be written as follows,

$$h(t) = \begin{cases} 
\frac{1}{\tau_r \tau_d} (e^{-\frac{t}{\tau_r}} - e^{-\frac{t}{\tau_d}}) & \text{if } t \geq 0 \\
0 & \text{otherwise}
\end{cases}$$

2) Tonic Component:

We model the tonic component as a summation of series of $P$ shifted and weighted cubic B-spline functions [19]. We can write it as follows,

$$y_s(t) = \psi(t) * q(t)$$

where $\psi(t)$ represents the cubic B-spline function and $q(t) = \sum_{j=0}^{P-1} q_j \delta(t - (j - 1)\Delta_s)$ is an abstraction of the tonic component coefficients representing the scaling and shifting of the cubic B-spline functions. $\Delta_s$ is the knot size of the cubic B-spline basis function which is an indicator of the smoothness of the tonic component. We select $\Delta_s = 6$ seconds for our analysis which is the same as the maximum value of the decay time that we allow in this study. We specifically select this value to allow a small increase in the tonic component during a phasic response [5]. We take $P = \frac{T_u}{\Delta_s} + 5$ for our analysis. Here we include 5 extra cubic spline waves to consider the tonic components outside the signal duration. As the tonic component is very slow in nature and also as we are modelling it with cubic B-spline waves, the tonic component for a time instance is dependent on the several neighboring coefficients $q_j$. To effectively describe the tonic component at the beginning and at the end of the signal we consider these extra 5 knots.

3) Discrete Model:

If the signal is periodically measured with an sampling period of $T_u$ for $M$ measurements, we can define the discrete observation equation for SC data $y_k$ as follows:

$$y_k = y_p(kT_u) + y_s(kT_u) + \nu_k$$

where $k = 1, 2, \ldots, M$ and $\nu_k$ is the measurement noise. We model $\nu_k$ as a Gaussian random variable. Using the sampled SC data $y_k$, we would like to estimate system parameters $\tau_r$ and $\tau_d$, and recover the tonic driver $q(t)$ and the neural stimuli $u(t)$ from ANS, i.e., amplitudes and timings of the impulses from ANS. The solution for $y_k$ is as follows:

$$y_k = a_k y_{p0} + b_k u + c_k q + \nu_k$$

where $a_k = e^{-\frac{kT_u}{\tau_r}}$, $b_k = \left[ h(kT_u) \ h(kT_u - T_u) \ \ldots \ h(T_u - \frac{N-1}{T_u}T_u) \right]^T$, $c_k = \left[ \psi(kT_u + \Delta_s) \ \psi(kT_u + \Delta_s) \ \ldots \ \psi(T_u - \Delta_s) \right]^T$ and $u = [u_1 \ u_2 \ \ldots \ u_N]^T$ represents a sparse vector (i.e., a limited number of the elements are non-zero) containing...
all the input neural stimuli amplitudes over the entire signal duration and \( q = [q_1, q_2, \ldots, q_N]^\top \) represents all the coefficients of the cubic B-spline basis functions; \( y_{p_0} \) represents the unknown initial value of the phasic component. Let \( y = [y_1, y_2, \ldots, y_M]^\top \), \( \tau = [\tau_1, \tau_2, \ldots, \tau_M]^\top \), \( \mathbf{A}_r = [a_1, a_2, \ldots, a_M]^\top \), \( \mathbf{B}_r = [b_1, b_2, \ldots, b_M]^\top \), \( \mathbf{C} = [c_1, c_2, \ldots, c_M]^\top \), and \( \nu = [\nu_1, \nu_2, \ldots, \nu_M]^\top \). Here, \( T_y \) is always an integer multiple of \( T_u \). Now the sampled data vector \( y \) is related to the sparse vector \( u \) representing the neural stimuli through the following equation:

\[
y = \mathbf{A}_r y_{p_0} + \mathbf{B}_r u + \mathbf{C} u + \nu.
\]  

(7)

\[C.\ \text{Estimation}
\]

We filter the signal using a low pass filter with a cut off frequency of 0.5 Hz to discard the high frequency noise as the SC signal is known to be band limited to 0.5 Hz [20], [12], [11]. Then, we downsample the filtered signal to achieve 2 Hz sampling frequency; hence, the sampling period for the SC signal \( T_y = 0.5 \) seconds. We let the sampling period for the neural stimuli \( T_u = 0.25 \) seconds to obtain a higher time resolution estimate of the input. In order to estimate the unknowns \( u, \tau \) and \( q \), using equation (7), we formulate the following optimization problem while assuming the sparsity constraint on \( u \) and including the constraint that the tonic component is always less than or equal to the SC signal (i.e., \( \mathbf{C} u \leq y \)):

\[
\min_{u, \tau, q} \sum_{\omega \leq \tau \leq \tau_{\text{max}}} J(u, \tau, q) = \frac{1}{2} ||y - \mathbf{A}_r y_{p_0} - \mathbf{B}_r u - \mathbf{C} u||_2^2 + \lambda_1 ||q||_2^2
\]

\[
\text{subject to } 0 \leq \tau \leq \tau_{\text{max}}
\]

where \( \tau_{\text{max}} \) and \( \tau_{\text{min}} \) are the upper and lower bound of the physiological system parameters. Here, we include the \( l_2 \)-norm penalization term with regularization parameter \( \lambda_1 \) to avoid over-fitting while solving for the tonic component coefficients \( q \). The above optimization formulation is a sparse recovery problem as \( ||u||_0 \ll M < N \). We encourage the sparsity for \( u \) with \( l_p \)-norm (\( 0 < p \leq 2 \)) regularization as a relaxation to the \( l_0 \)-norm. We can re-write the optimization problem as follows:

\[
\min_{u, \tau, q} \sum_{\omega \leq \tau \leq \tau_{\text{max}}} J(u, \tau, q) = \frac{1}{2} ||y - \mathbf{A}_r y_{p_0} - \mathbf{B}_r u - \mathbf{C} q||_2^2 + \lambda_1 ||q||_2^2
\]

\[+ \lambda_2 ||u||_p^p \]

(9)

We can solve the inverse problem of finding a nonnegative \( u \) in (9) with a specific sparsity level using the iterative least squares (IRLS) approach Focal Underdetermined System Solver (FOCUS+) algorithm [21]. In each iteration of the IRLS algorithm, we use GCV for estimating an appropriate regularization parameter \( \lambda_1 \) similar to [15], [14], [10], [11]. We also use GCV to obtain \( \lambda_2 \) similar to [22].

In order to solve for the physiological system parameters \( \tau \) and the initial phasic SC condition \( y_{p_0} \), we solve the optimization problem in (9) using the interior-point method. We concatenate \( y_{p_0} \) with \( \tau \) to consider it as a third parameter and define \( \theta = [\tau^\top, y_{p_0}]^\top \). To estimate all the unknowns, we implement the following block coordinate descent algorithm using the optimization formulation in (9):

\[\text{Algorithm: Generalized-Cross-Validation-Based Block Coordinate Descent}\]

(a) Let \( j = 0 \). Initialize \( \theta^0 \) by sampling a uniform random variable on \([0.10, 1.5]\) for \( \tau_r(0) \), on \([1.5, 6]\) for \( \tau_q(0) \), and on \([0, 6]\) for \( y_{p_0} \); also initialize \( q^0 \) by sampling \( P \) number of Gaussian random variables with mean 0.1 and standard deviation of 0.02.

(b) Set \( j = j + 1 \).

(c) Set \( \theta = \theta^{(j-1)} \) and \( q = q^{(j-1)} \); use FOCUSS+ [21] to solve the inverse problem in (9) to find the stimuli \( \hat{u}^{(j)} \) by initializing \( \hat{u}^{(j-1)} \) at a vector of all ones.

(d) Set \( u = \hat{u}^{(j)} \) and \( q = \hat{q}^{(j-1)} \); use the interior point method to minimize the optimization problem in (9) to solve for \( \hat{\theta}^{(j)} \) by initializing the optimization problem at \( \hat{\theta}^{(j-1)} \).

(e) Set \( \theta = \hat{\theta}^{(j)} \) and \( u = \hat{u}^{(j)} \); use the interior point method and minimize the optimization problem in (9) to solve for \( \hat{q}^{(j)} \) by initializing the optimization problem at \( \hat{q}^{(j-1)} \).

(f) Repeat between steps (b)-(e) until \( j = 30 \).

(g) Let \( i = 0 \). Set \( \theta^0 = \hat{\theta}^{(j)} \), \( u^0 = \hat{u}^{(j)} \), and \( q^0 = \hat{q}^{(j)} \).

(h) Set \( i = i + 1 \).

(i) Set \( \theta = \hat{\theta}^{(i-1)} \) and \( q = \hat{q}^{(i-1)} \); use GCV-FOCUS+ [23] to solve the inverse problem in (9) to find the stimuli \( \hat{u}^{(i)} \) by initializing at \( \hat{u}^{(i-1)} \).

(j) Set \( u = \hat{u}^{(i)} \) and \( q = \hat{q}^{(i-1)} \); use the interior point method to minimize the optimization problem in (9) to solve for \( \hat{\theta}^{(i)} \) by initializing at \( \hat{\theta}^{(i-1)} \).

(k) Set \( \theta = \hat{\theta}^{(i-1)} \) and \( u = \hat{u}^{(i-1)} \); use GCV [22] to obtain \( \lambda_2 \), and use the interior point method to minimize the optimization problem in (9) to solve for \( \hat{q}^{(i)} \) by initializing at \( \hat{q}^{(i-1)} \).

(l) Iterate between (h)-(k) until convergence.

We run the algorithm for several random initial values of system parameters. Finally, we choose the estimated values that minimize \( ||y - \mathbf{A}_r y_{p_0} - \mathbf{B}_r u - \mathbf{C} q||_2^2 \).

III. RESULTS

Using the proposed approach, we decompose the SC measurements collected during an auditory stimulation experiment and separate the tonic and phasic components. Furthermore, we recover the underlying stimuli \( u(t) \), the corresponding rise time \( \tau_r \), decay times \( \tau_q \), and the initial phasic SC condition \( y_{p_0} \). Results in Figure 1 show that the proposed algorithm successfully estimates the tonic and phasic components along with the timings and amplitudes of neural stimuli for two female participants (subject ID: 12 and
Fig. 1. Estimated Decomposition of the Experimental SC Signals for Two Female and Two Male Participants: In each of the panels, i) the top sub-panel shows the experimental SC signal (blue stars), the reconstructed SC signal (red curve), the estimated tonic component (green curve), and the timings of the auditory stimulations (gray vertical lines); ii) the bottom sub-panel shows the estimated phasic component (blue curve), estimated neural stimuli timings and amplitudes (black vertical lines) due to ANS activation and the timings of the auditory stimuli (gray vertical lines).

Fig. 2. Performance Comparison of Proposed Approach with Existing Approaches for Simulated Data: Each panel shows the decomposition performance based on simulated SC signal with 25 dB noise. The top, middle, and bottom panel shows the result using LedaLab [7], cvxEDA [9], and our proposed approach, respectively. In each panel, blue stars represent the simulated data, red vertical lines represent the ground truth neural stimuli, black vertical lines represent the recovered neural stimuli, the green curve represents the tonic component, the black dotted curve represents the ground truth for tonic component, and the red curve represents the reconstructed signal.

The multiple correlation coefficient ($R^2$) has been calculated for all the four reconstructed signals. The $R^2$ values are 0.9953, 0.9964, 0.9921 and 0.9926, respectively. The high values of $R^2$ SC data suggest that our proposed algorithm can successfully uncover physiologically plausible ANS stimulation and separate the tonic and phasic components. The run times for deconvolution are 455.73 seconds, 971.15 seconds, 1186.3 seconds and 994.91 seconds, respectively.

To verify our approach and to compare our method with the other existing approaches, we use synthetic simulated data. We use the synthetic neural stimulation, the physiological parameters, and the cubic-spline coefficients to simulate the data. The physiological parameters are chosen to be 0.7 seconds for the rise time and 2 seconds for the decay time based on the assumption in previous studies [9], [7], [5]. We add Gaussian random noise with 25 dB SNR with respect to the phasic component. Figure 2 shows the decomposition of tonic, phasic component, and recovered neural stimuli using LedaLab [7], cvxEDA [9], and our proposed approach. The results show our algorithm is outperforming other the existing algorithm. As observed in Figure 2, LedaLab and cvxEDA are providing less sparse solutions compared to the ground truth. Some of the pulses detected by these algorithms are capturing noise. On the other hand, our proposed approach is performing well in balancing between the sparsity level and discarding noise. Furthermore, the previous approaches do not perform deconvolution to be able to recover the system parameters. In running the LedaLab and cvxEDA algorithms, we set the system parameters to the values used for simulating the data. In our approach, we are able to recover the system parameters; hence, we estimate the system parameters and obtain the rise and decay times with errors of 1.14% and 7.83%, respectively.

IV. DISCUSSIONS

Decomposition of SC signals and identifying the neural stimuli along with the rise and decay times of the SC responses is a challenging problem. The optimization formu-
ation in (8) has many degrees of freedom which could result in identifiability problems. There exist many solutions for the unknowns that can closely approximate the sampled signal. However, incorporating sufficient physiologically plausible constraints can make this problem more tractable. Firstly, we consider the sparsity constraint on the neural stimuli. Secondly, we constrain the physiological system parameters within physiologically feasible bounds ($\tau_{\min} = [0.10 \ 1.5]^T$ and $\tau_{\max} = [1.5 \ 6]^T$) [11]. We also impose constraints on the smoothness of the cubic B-spline basis function by including $l_2$-norm penalization. Finally, we incorporate the GCV technique [22] to have appropriate estimates of $\lambda_1$ and $\lambda_2$ to achieve a balance between capturing the data and residual error.

The proposed optimization problem is non-convex in terms of physiological system parameters. It is possible to have a solution that stagnates at a local minima. Also, the tonic component of the SC data might be captured in the phasic component and the solution for the rise or decay time could stagnate in the boundary. To account for that, we initialize the optimization problem with several random initial values of physiological parameters and the tonic component. We discard all the solutions that stagnated to the boundary. Among the rest, we take the one that minimizes the residual error.

V. CONCLUSION AND FUTURE WORK

In this study, we have shown a promising method for decomposition of a SC signal by modelling tonic and phasic components and formulating an optimization problem to recover the unknowns with physiologically plausible constraints. We have proposed a block coordinate descent approach to recover the unknowns by incorporating sparse recovery for neural stimuli, interior-point method for physiological system parameter and tonic component estimation. Finally, we have implemented GCV to obtain regularization parameters for both $l_1$-norm and $l_2$-norm penalization terms in the optimization problem to avoid over-fitting. Both experimental and simulated data show that our approach outperforms previous methods.

The tonic component of SC might vary in different regions of skin, however, the phasic component is modulated by the same sparse neural stimuli from ANS [12]. This characteristic of SC can be exploited to obtain a better decomposition strategy using multi-channel SC data. As future directions, we plan to develop fast decomposition schemes for tonic and phasic components from single and multi-channel SC data to obtain more reliable estimate for real-time applications.

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